Threshold voltage model for small geometry AlGaN/GaN HEMTs based on analytical solution of 3-D Poisson’s equation

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Abstract

A simple and accurate analytical model for the threshold voltage of AlGaN/GaN high electron mobility transistor (HEMT) is developed by solving three-dimensional (3-D) Poisson equation to investigate the short channel effects (SCEs) and the narrow width effects present simultaneously in a small geometry device. It has been demonstrated that the proposed model correctly predicts the potential and electric field distribution along the channel. In the proposed model, the effect of important parameters such as the thickness of the barrier layer and its doping on the threshold voltage has also been included. The model is, further, extended to find an expression for the threshold voltage in the sub-micrometer regime. The accuracy of the proposed analytical model is verified by comparing the model results with 3-D device simulations for different gate lengths and widths.

Keywords: AlGaN/GaN HEMT; Small geometry; Short channel effects; Three-dimensional (3-D) modeling; Threshold voltage

1. Introduction

High electron mobility transistors (HEMTs) are extremely promising devices in the area of high speed ICs [1,2] for optical communication systems and in microwave analog circuits for mobile communication systems [3,4]. AlGaN/GaN HEMTs, in particular, are emerging as excellent candidates for their potential use at microwave frequencies [2] because of the material properties of GaN such as high peak electron velocity, saturation velocity and thermal stability. It is a trend in compound semiconductor technology to continuously develop devices which are faster, smaller and consume less power for the same level of integration [5]. Enhancement in device integration technology requires rapid scaling of device dimensions. Small signal HEMTs typically has gate lengths below 0.15 \( \mu \text{m} \) [6]. Recently, AlGaN/GaN HEMT with gate length 0.09 \( \mu \text{m} \) for operation in 30 GHz range has also been reported [7]. For mass production of AlGaN/GaN HEMTs, the device dimensions are continuously being scaled down to sub-100 nm regime. AlGaN/GaN HEMT with gate width of 15 \( \mu \text{m} \) has also been successfully fabricated [8]. However, as the device dimensions are reduced to sub-micron level, it becomes more challenging to develop an analytical model for the same. With the reduction in device dimensions, two- and three-dimensional electrostatic effects tend to degrade the device performance [9] in terms of the threshold voltage, a key parameter in the design of HEMT circuits.

Short channel effects (SCE) in small geometry transistors have received considerable attention in the recent years because of push towards the smaller devices for faster and denser integrated circuits. Solution of the 2-D Poisson equation for HEMTs has been obtained using various approaches [10–13]. However, continuous device scaling is bringing HEMTs to the regime of short channel (for lower supply voltage and high-speed operations) and narrow width (for lower power consumption and higher density) HEMTs. Such devices, called the small geometry HEMTs,
are quite complex as 3-D effects start affecting the performance of the small geometry FETs. Although the device modeling of SCEs is fairly mature, the modeling of small geometry HEMTs has not been done so far, even though the narrow width effects become predominant as the device width is reduced. This is because 3-D nature of the problem makes the analysis very difficult and time consuming. Hence, an accurate 3-D analytical model is needed to predict the sub-threshold behavior of small geometry HEMTs which simultaneously incorporates both the SCEs and narrow width effects.

In this work, a simple and accurate threshold voltage analytical model for small geometry HEMTs is developed by solving the 3-D Poisson equation using the standard separation of variables technique [14]. Section 2 gives the 3-D potential and electric field analysis of the proposed model. The expression for the threshold voltage is obtained in Section 3. The model which simultaneously takes into account both the SCEs and narrow width effects, is validated with simulation results obtained from ATLAS 3-D device simulator [15] in Section 4. Section 5 finally concludes the paper.

2. Model formulation

2.1. Three-dimensional potential analysis

Fig. 1(a) shows the schematic structure of AlGaN/GaN HEMT where source and drain electrodes are ohmic contacts while the gate electrode, which modulates charge in the conducting channel, is a Schottky barrier placed on n-AlGaN layer. Source and drain are modeled as heavily doped regions with source—AlGaN and drain—AlGaN junction located at \( y = 0 \) and \( L_{\text{eff}} \), respectively, where \( L_{\text{eff}} \) is the effective channel length. The front interface and the back heterointerfaces are located at \( x = 0 \) and \( d \), respectively, where \( d = d_d + d_s \), \( d_s \) is the thickness of the spacer layer and \( d_d \) is the thickness of the AlGaN spacer layer. The sidewall interfaces are located at \( z = 0 \) and \( W \) and \( V_g \) is the applied gate potential. Fig. 1(b) shows the band diagram of the structure under the influence of the Schottky gate in contact with n-AlGaN layer. The barrier layer is fully depleted under the normal operating conditions and the electrons are confined to the heterointerface. These conducting electrons form a two-dimensional electron gas (2-DEG) that lies in a triangular potential well at the heterointerface of the wide bandgap AlGaN and narrow bandgap GaN layer.

In the sub-threshold region, since the currents are small, Poisson equation alone is adequate to analyze the structure shown in Fig. 1. Assuming that the impurity density in the channel region is uniform and that the region under the gate is fully depleted under normal operating conditions, partly due to Schottky barrier at the surface and partly due to electron diffusion into the channel at the heterointerface, the 3-D Poisson

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**Nomenclature**

- \( V_{\text{bi}} \) built-in potential
- \( \phi(x,y,z) \) potential at any point \( (x,y,z) \) in the channel
- \( \phi_b \) Fermi potential
- \( V_{\text{fb}} \) flat band voltage
- \( N_d \) doping concentration of AlGaN layer
- \( E_{\text{int}} \) field at the heterointerface
- \( D_d \) thickness of doped AlGaN layer
- \( D_i \) thickness of the spacer layer
- \( L_{\text{eff}} \) effective channel length
- \( W \) effective channel width

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Fig. 1. (a) Schematic diagram of FD Al_{0.2}Ga_{0.8}N/GaN HEMT. (b) The band diagram of AlGaN/gaN heterostructure in the 2-DEG control regime by Schottky gate for \( V_{\text{gs}} = 0.1 \) and 1.0 V.
equation can be expressed as
\[
\frac{\partial^2 \phi(x, y, z)}{\partial x^2} + \frac{\partial^2 \phi(x, y, z)}{\partial y^2} + \frac{\partial^2 \phi(x, y, z)}{\partial z^2} = -\frac{qN_d}{\varepsilon_a}
\]
(1)
where \(\phi(x, y, z)\) is the potential at any point \((x, y, z)\) in the channel, \(N_d\) is the doping concentration of \(\text{Al}_{0.2}\text{Ga}_{0.8}\text{N}\) layer, \(q\) is the electronic charge, \(\varepsilon_a\) is the \(\text{Al}_{0.2}\text{Ga}_{0.8}\text{N}\) dielectric constant. The Poisson equation is solved using the following boundary conditions:
\[
\phi(0, y, z) = V_g - V_{fb}
\]
(2)
\[
\frac{\partial \phi(d, y, z)}{\partial x} = -E_{\text{int}}
\]
(3)
\[
\phi(x, 0, z) = V_{bi}
\]
(4)
\[
\phi(x, L_{\text{eff}}, z) = V_{bi} + V_{ds}
\]
(5)
\[
\phi(x, y, 0) = V_g - V_{fb}
\]
(6)
\[
\phi(x, y, W) = V_g - V_{fb}
\]
(7)

The boundary condition given by (2) takes the metal gate of length \(L_{\text{eff}}\) and width \(W\) as an equipotential plane and \(V_{fb} = \phi_{m} - \phi_s\) is the flat band voltage, while (3) gives the field at the heterointerface \[11\]. The boundary conditions at the source and the drain ends of the channel are given by (4) and (5), \(V_{bi}\) being the built-in potential \[16\] and \(V_{ds}\) is the applied drain to source voltage; while (6) and (7) give the potential at the edges of the transistor as the applied gate potential.

The 3-D Poisson equation is solved by separating it into 1-D Poisson equation and 2-D and 3-D Laplace equation as
\[
\frac{d^2 \phi_{1-D}(x)}{dx^2} = -\frac{qN_d(x)}{\varepsilon_a}
\]
(8)
\[
\frac{\partial^2 \phi_{2-D}(x, y)}{\partial x^2} + \frac{\partial^2 \phi_{2-D}(x, y)}{\partial y^2} = 0
\]
(9)
\[
\frac{\partial^2 \phi_{3-D}(x, y, z)}{\partial x^2} + \frac{\partial^2 \phi_{3-D}(x, y, z)}{\partial y^2} + \frac{\partial^2 \phi_{3-D}(x, y, z)}{\partial z^2} = 0
\]
(10)
where
\[
\phi(x, y, z) = \phi_{1-D}(x) + \phi_{2-D}(x, y) + \phi_{3-D}(x, y, z)
\]
(11)
\(\phi_{1-D}(x)\) is the solution of (8) for the boundary condition
\[
\phi_{1-D}(x)|_{x=0} = V_g - V_{fb}
\]
(12)
\[
\left.\frac{d\phi_{1-D}(x)}{dx}\right|_{x=d} = -E_{\text{int}}
\]
(13)
\(\phi_{2-D}(x, y)\) is the solution of (9) for the boundary conditions
\[
\phi_{2-D}(0, y) = 0
\]
(14)
\[
\frac{d\phi_{2-D}(d, y)}{dy} = 0
\]
(15)
\[
\phi_{2-D}(x, 0) = V_{bi} - \phi_{1-D}(x)
\]
(16)
\[
\phi_{2-D}(x, L_{\text{eff}}) = V_{bi} - \phi_{1-D}(x) + V_{ds}
\]
(17)
and \(\phi_{3-D}(x, y, z)\) is the solution of (10) for the boundary conditions
\[
\phi_{3-D}(0, y, z) = 0
\]
(18)
\[
\phi_{3-D}(x, 0, z) = 0
\]
(19)
\[
\phi_{3-D}(x, L_{\text{eff}}, z) = 0
\]
(20)
\[
\phi_{3-D}(x, y, 0) = V_g - V_{fb} - \phi_{1-D}(x) - \phi_{2-D}(x, y)
\]
(21)
\[
\phi_{3-D}(x, y, W) = V_g - V_{fb} - \phi_{1-D}(x) - \phi_{2-D}(x, y)
\]
(22)
\[
\phi_{3-D}(x, y, z) = \phi_{1-D}(x) + E_{\text{int}}(d - x) - \frac{qN_d(x - d_a)^2}{2\varepsilon_a}
\]
(24)
where
\[
\phi_{1-D}\text{ int} = \phi_{1-D}(d)
\]
and
\[
E_{\text{int}} = \left.\frac{-\partial \phi_{1-D}}{\partial x}\right|_{x=d}
\]
(11)
2.1.1. Solution of one-dimensional potential, \(\phi_{1-D}(x)\)
\(\phi_{1-D}(x)\) is of the form
\[
\phi_{1-D}(x) = \phi_{1-D}\text{ int} + E_{\text{int}}(d - x) - \frac{qN_d(x - d_a)^2}{2\varepsilon_a}
\]
(24)
where
\[
\phi_{1-D}\text{ int} = \phi_{1-D}(d)
\]
and
\[
E_{\text{int}} = \left.\frac{-\partial \phi_{1-D}}{\partial x}\right|_{x=d}
\]
(11)
2.1.2. Solution of two-dimensional potential, \(\phi_{2-D}(x, y)\)
\(\phi_{2-D}(x, y)\) is the solution of (9) for the boundary conditions (14)–(17) obtained by using separation of variable method \[14\].
\[
\phi_{2-D}(x, y) = \sum_{r=1}^{\infty} \frac{1}{\sinh(\beta_r L_{\text{eff}})} \left[V' \sinh(\beta_r y) + V V' \sinh(\beta_r (L_{\text{eff}} - y))\right] \sinh(\beta_r y)
\]
(25a)
where \(\beta_r\) is given by
\[
\beta_r = (2r - 1) \frac{\pi}{2d}
\]
(25b)
and \(r\) varies from 1 to \(\infty\)
\[
V = \frac{i_{d1} + (qN_d/2\varepsilon_a)(\cos(\beta_r d/\beta_r)dd_a - d^2) + (qN_d/\varepsilon_a)(\sin(\beta_r d/\beta_r)^2)(d - d_a)}{i_{d1}}
\]
(25c)
$$V' = V1 + \frac{V_d((1 - \cos(\beta,d))/\beta_r)}{i_{d1}}$$  \hfill (25d) \\
$$i_{d1} = \frac{1}{4\beta_r}[2\beta_r d - \sin(2\beta_r d)]$$  \hfill (25e) \\
$$i_{dn1} = \frac{1}{\beta_r} \left[ \frac{V_{bi} - \varphi_{1-D \text{ int}} - E_{int}d + \frac{q}{2\varepsilon_a} N_{d}dd_d}{\beta_r} \right]$$  \\
$$- \left[ V_{bi} - \varphi_{1-D \text{ int}} \right] \frac{\cos(\beta_r d)}{\beta_r} + E_{int} \sin(\beta_r d)$$  \\
$$+ \frac{qN_{d} \cos(\beta_r d)}{\varepsilon_a}$$  \hfill (25f) \\

2.1.3. Solution of three-dimensional potential, $\varphi_{3-D}(x, y, z)$

The solution of 3-D Laplace equation (10) is obtained using the boundary conditions (18)–(23). Using the separation of variable technique, solution of (10) is of the form

$$\varphi_{3-D}(x, y, z) = [A \sin(xy) + B \cos(xy)][C \sin(\beta x) + D \cos(\beta x)]$$  \\
$$\times [E \sinh(\gamma z) + F \cosh(\gamma z)]$$  \hfill (26) \\

where $A, B, C, D, E, F, \alpha, \beta, \gamma$ are arbitrary constants and

$$\alpha^2 + \beta^2 = \gamma^2$$  \hfill (27) \\

From the boundary conditions (18) and (19), we get

$$\beta_r = (2r - 1) \frac{\pi}{2d}$$  \hfill (28) \\

where $r$ varies from 1 to $\infty$ and from (20) and (21), we get

$$\alpha_s = \frac{\pi}{L_{\text{eff}}}$$  \hfill (29) \\

where $s$ varies from 1 to $\infty$. Thus, the solution is of the form

$$\varphi_{3-D}(x, y, z) = \sum_{1}^{\infty} \sum_{1}^{\infty} [P_{sr} \cosh(\gamma_{sr}z) + Q_{sr} \sinh(\gamma_{sr} z)]$$  \\
$$\times \sin(\beta_r x) \frac{\sin(\alpha_s(z - L_{\text{eff}}))}{\cos(\alpha_s L_{\text{eff}})}$$  \hfill (30) \\

and

$$\gamma_{s}^2 + \beta_r^2 = \gamma_{sr}^2$$  \hfill (31) \\

Using the boundary condition (22) and multiplying both sides by $\sin(\beta_r x)(\sin(\alpha_s(y-L_{\text{eff}}))/\cos(\alpha_s L_{\text{eff}}))$ to satisfy the condition of orthogonality and integrating with respect to $x$ from “0” to “d” and with respect to $y$ from “0” to “L_{\text{eff}}”, we get

$$P_{sr} = \frac{I_{3dn}}{I_{3dd}}$$  \hfill (32) \\

where

$$I_{3dd} = \frac{1}{2\cos^2(\alpha_s L_{\text{eff}})} \left[ L_{\text{eff}} - \frac{\sin(2\alpha_s L_{\text{eff}})}{2\alpha_s} \right] \frac{1}{4\beta_r} (2\beta_r d - \sin(2\beta_r d))$$  \hfill (33) \\

$$I_{3dn} = \frac{\cos(\alpha_s L_{\text{eff}}) - 1}{\frac{qN_{d} \cos(\alpha_s L_{\text{eff}})}{2\varepsilon_a} \cos(\beta_r d(2d - d_d))}$$  \\
$$- \frac{qN_{d} \sin(\beta_r d(d - d_d))}{\varepsilon_a}$$  \\
$$- \frac{1}{\cos(\alpha_s L_{\text{eff}})} \sum_{r = 1}^{\infty} \frac{i_2}{\sinh(\beta_r L_{\text{eff}})} [V' i_3 + V' i_4]$$  \hfill (34) \\

where $V'$ and $V_1$ are given by (25c) and (25d) and

$$i_1 = \left[ V_g - V_{fb} - \varphi_{1-D \text{ int}} - E_{int}d + \frac{q}{2\varepsilon_a} N_{d}dd_d \right] \frac{1}{\beta_r}$$  \\
$$- \left[ V_g - V_{fb} - \varphi \right] \frac{\cos(\beta_r d)}{\beta_r}$$  \\
$$+ E_{int} \sin(\beta_r d) + \frac{qN_{d} \cos(\beta_r d)}{\varepsilon_a}$$  \hfill (35) \\

$$i_2 = \frac{1}{4\beta_r} [2\beta_r d - \sin(2\beta_r d)]$$  \hfill (36) \\

$$i_3 = (\frac{\sin(\alpha_s L_{\text{eff}})/\beta_r}{(1 + (\gamma_{sr}/\beta_r^2))})$$  \hfill (37) \\

$$i_4 = (\frac{\cos(\alpha_s L_{\text{eff}}) \sinh(\gamma L_{\text{eff}}) - \sin(\alpha_s L_{\text{eff}}) \cosh(\gamma L_{\text{eff}}) / \beta_r}{1 + (\gamma_{sr}/\beta_r^2)})$$  \hfill (38) \\

On applying the boundary condition (23) and multiplying both sides by $\sin(\beta_r x)(\sin(\alpha_s(y-L_{\text{eff}}))/\cos(\alpha_s L_{\text{eff}}))$ and integrating w.r.t. $x$ from “0” to “d” and w.r.t. to $y$ from “0” to “L_{\text{eff}}”, we get

$$P_{sr} = \frac{Q_{sr} \sinh(\gamma_{sr} W)}{1 - \cosh(\gamma_{sr} W)}$$  \hfill (39) \\

where

$$Q_{sr} = \frac{I_{3dn}}{I_{3dd}} \frac{1 - \cosh(\gamma_{sr} W)}{\sinh(\gamma_{sr} W)}$$  \hfill (40) \\

Substituting (39) in (30) we get

$$\varphi_{3-D}(x, y, z) = \sum_{1}^{\infty} \sum_{1}^{\infty} R_{sr} [\sinh(\gamma_{sr} (W - z))$$  \\
$$+ \sinh(\gamma_{sr} z)] \frac{\sin(\alpha_s(y - L_{\text{eff}}))}{\cos(\alpha_s L_{\text{eff}})}$$  \hfill (41) \\

where

$$R_{sr} = \frac{Q_{sr}}{1 - \cosh(\gamma_{sr} W)}$$  \hfill (42) \\

Substituting (24), (25) and (40) in (11), we get potential at any point $(x, y, z)$ in the channel in terms of $\varphi_{1-D \text{ int}}$ and $E_{int}$. $\varphi(x, y, z)$ can be obtained in terms of $V_g$ by substituting (24) in (12) and (13) and using

$$E_{int} = \left( \frac{\varphi_{1-D \text{ int}}}{d} + \frac{qN_{d}dd_d}{2\varepsilon_a} \right)$$  \hfill (43)
Thus, using
\[ V_g = V_{th} \]

where \( \varphi_b \) is the Fermi potential, in (47), we get the expression for threshold voltage of the small geometry device as
\[ V_{th} = V_{thL} - V_{thS} - V_{thV} \]  

(48)

where
\[ V_{thL} = 2\varphi_b + E_{int} - \frac{qN_ad^4}{2\varepsilon_a} + \varphi_{th} \]

(49)

\[ V_{thS} = \frac{1}{\sinh(\beta y_{min})} [V' \sinh(\beta y_{min}) + V_1 \sinh(\beta(L_{eff} - y_{min}))] \]

(50)

\[ V_{thV} = R \left[ 2 \sinh \left( \frac{W}{2} \right) \frac{\sin(\gamma(\gamma_L - L_{eff}))}{\cos(\gamma L_{eff})} \right] \]

(51)

\( V_{thL} \) is independent of length and width of the device and is the threshold voltage of a long and wide channel HEMT. It is \( V_{thS} \) which depends on the drain to source voltage and the effective channel length but is independent of the width of the channel. Thus, this term is responsible for the reduction in the threshold voltage, \( V_{th} \) due to the short channel length. It is \( V_{thV} \) which depends on the width of the channel and thus affects the threshold voltage as the width is reduced.

4. Results and discussion

The analytical model developed has been verified by comparing the results obtained by ATLAS 3-D device simulator. Devices with gate length 0.09, 0.12, 0.15 and 0.25 \( \mu \)m have been simulated keeping all other parameters same. The device parameters used are as follows: \( d_2 = 25 \) nm, \( d_1 = 5 \) nm, \( N_d = 1 \times 10^{24} \) m\(^{-3} \), \( m = 20\% \), \( W = 1.5 \) \( \mu \)m.

Based on the analytical model and the ATLAS-3D simulation results, Fig. 2 shows the variation of channel potential with the normalized channel position (\( y/L_{eff} \)) for two different channel lengths. It can be seen that for the same drain bias, \( V_{ds} = 0.1 \) V, the minimum channel potential \( \varphi_{c, min} \) increases as the channel length decreases because of the SCEs and hence the channel barrier is reduced. Fig. 2 inset shows the effect of thickness of AlGaN layer on the channel potential. An increase in the thickness increases the DIBL effect. The modeled results are compared with the simulation results and a good agreement is obtained between the two.

Fig. 3 shows the variation of channel potential with the normalized channel position for three different drain voltages for a gate length 0.12 \( \mu \)m. The plot shows that as \( V_{ds} \) increases, the channel potential minimum increases reducing the source barrier, thus leading to an increase in current. It also shifts the point of minimum potential...
towards the source [10]. The calculated results are compared with the ATLAS 3-D simulations and a good agreement is obtained.

Fig. 2. Channel potential variation with normalized position along the channel length for two different channel lengths at $z = W/2$, $V_{ds} = 0.1$ V and $d = 30$ nm for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT. Inset: Channel potential variation with normalized position along the channel length for two different $d_d$ at $z = W/2$. $L_{\text{eff}} = 0.09$ $\mu$m for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT.

Fig. 3. Channel potential variation with normalized position along the channel length for three different drain biases at $z = W/2$, $L_{\text{eff}} = 0.12$ $\mu$m and $d = 30$ nm for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT.

Fig. 4. Electric field variation with normalized position along the channel length for different drain biases and $d_d = 25$ nm for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT. Inset: Electric field variation with normalized position along the channel length for $V_{ds} = 0.1$ V, $L_{\text{eff}} = 0.09$ $\mu$m, $W = 1.5$ $\mu$m and $d_d = 30$ nm for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT.

Fig. 5. Channel potential variation with normalized position along the channel width at the heterointerface for various channel widths for Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT.

Fig. 4 shows the lateral electric field distribution along the channel at two different drain voltages. Fig. 4 inset shows the lateral electric field distribution along the channel for $L_{\text{eff}} = 0.09$ $\mu$m and $d_d = 30$ nm. The analytical results agree well with the simulation results except near the gate edges where the discrepancy is due to using finite terms in our calculations.

Fig. 5 shows the variation of channel potential as a function of channel width for channel length 1.5 $\mu$m. The potential variation is symmetrical about $z = W/2$. For longer widths, the channel potential is constant for most of
the width showing a broad minimum. On the other hand, as the $W/L_{\text{eff}}$ ratio is reduced the inverse narrow width effects start to appear and for narrow widths, the channel potential shows a sharp minimum at $z = W/2$ which is higher than the value for longer width device (similar to the SCEs) because of increased proximity of the sidewall gate as the width reduces. However, for significant applied drain bias, the SCEs overshadow the narrow width effects for typical values of $W/L_{\text{eff}} (> 1)$ used in digital circuits. Hence, to highlight the narrow width effects, gate length has been taken to be more than the channel width.

**Fig. 6** gives the variation of minimum channel potential with the gate length. The minimum channel potential increases with decrease is channel length because of the increased field penetration from the source to drain. Increase in minimum channel potential with decreasing length is observed for gate lengths less than 0.15 $\mu$m.

**Fig. 7** gives the variation of threshold voltage with the channel length for two different values of ‘‘$d_d$’’. As can be seen from the figure, threshold voltage varies little for long gate length devices. This is apparent from Eq. (50) where $\sinh(\beta L_{\text{eff}})$ is in the denominator and hence, this term will increase with decrease in $L_{\text{eff}}$. But for large value of $L_{\text{eff}}$, $\sinh(\beta L_{\text{eff}})$ becomes large and this term goes to zero and the threshold voltage corresponds to that of a long channel HEMT. The model correctly predicts the threshold voltage roll-off at shorter channel lengths which has also been validated through the simulations for $d_d = 25$ nm. A close match with the simulated results proves the accuracy of the analytical model.

**Fig. 8** shows the variation of the threshold voltage of a short channel Al$_{0.2}$Ga$_{0.8}$N/GaN HEMT as a function of drain voltage for channel lengths 0.12, 0.2 and 0.3 $\mu$m with $V_{\text{ds}}$ varying from 0 to 2 V, obtained analytically. It is seen that the threshold voltage swings from $-0.27$ to $-0.34$ V for $V_{\text{ds}}$ varying from 0 to 2 V for $L_{\text{eff}} = 0.12$ $\mu$m while the shift for $L_{\text{eff}} = 0.2$ and 0.3 $\mu$m is negligible. This SCE is due to the penetration of junction electric field in the channel region, causing barrier lowering which leads to reduction in threshold voltage. Model correctly predicts the DIBL at shorter channel lengths.

**Fig. 9** gives the variation of threshold voltage of short geometry AlGaN/GaN HEMT with channel width; obtained using our analytical model for $L = 1.5$ $\mu$m. Again the length has been taken to be more than the width to highlight the narrow width effects which get overshadowed very easily by the SCEs at smaller lengths. For large $W$, ‘‘$P$’’ goes to zero and there is no dependence of the
threshold voltage on $W$. However, for smaller values of $W$, “$W$” dependent term in Eq. (48) becomes a positive quantity which increases with decrease in “$W$” and thus results in threshold voltage roll-off.

5. Conclusion

An analytical 3-D threshold voltage model of fully depleted AlGaN/GaN HEMT is presented and closed-form expressions for electrostatic potential, minimum channel potential and threshold voltage are obtained. The model also permits us to conclude that both the short channel and narrow width effects are more for larger $N_d$ and larger thickness $d$ of AlGaN layer. This model is useful for the analysis of small geometry HEMTs as it takes into account both the SCEs due to reduced length and narrow width effects due to reduced width for applications in wireless communication (handsets). To the author’s knowledge, there is no such model available for HEMTs which takes into account both the effects simultaneously. A good overall match of the new model with simulation results proves the validity of the model.

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